



CFD analysis of nonlinear processes in pulse tube refrigerators: Streaming induced by vortices

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ABSTRACT

Streaming in the pulse tube refrigerator is a crucial nonlinear flow and heat transfer phenomenon which considerably affects the refrigeration temperature and performance. The third type streaming in the pulse tube refrigerator is studied using computational fluid dynamics method for the first time. A two-dimensional simulation of an inline inertance tube pulse tube refrigerator (ITPTR) is performed for different operating frequencies with the help of the FLUENT[®] package. The streaming is found to be formed due to the generation, evolution and shedding of vortices and pressure drops which are induced by the hydrodynamic and thermodynamic asymmetries along the refrigeration system. The pressure drops due to abrupt changes of the tube cross sections at both hot and cold ends in the pulse tube are calculated and the mass flow rate of the streaming is predicted. The geometry, temperature gradient and especially frequency are revealed as the main factors influencing the streaming patterns and final refrigeration performance. The numerical results agree well with the substantial experiments and indicate further suppression and optimization methods.

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1. Introduction

The pulse tube refrigerator (PTR) without any moving part under cryogenic temperature is promising for many high-reliability applications such as aerospace, superconducting electronics, magnetic resonance imaging and helium liquefaction [1]. The working gas inside PTR is characterized as the alternating current (AC) gas flow, which moves alternatively backward and forward with pulsating pressure at certain frequencies so as to form a desired temperature gradient in the regenerator and pulse tube. The inside oscillating flow executes a thermodynamic cycle that is similar to the Stirling cycle in the regenerator. Therefore, the PTR can offer efficiency which is comparable to that of conventional refrigerators. Until recently, cryocooler performance and reliability have been continually improving with both novel configurations and better understanding of the flow physics and heat transfer within the PTR [2]. Among those developments, discovering and suppressing the acoustic streaming or so-called direct current (DC) gas flow in PTR and other thermoacoustic engines [3] is one of the most important one. Since Gedeon flow [4] was discovered in PTR, the important theoretical and application value

of streaming in PTR has been well recognized because of its detrimental effect on refrigeration temperature and reliability.

A key point for Gedeon streaming in PTR or Stirling cryocooler is the existence of a closed-loop path for steady streaming, which carries a nonzero mass flow through a cross section of a thermoacoustic resonator. For PTR it only occurs in the double-inlet version. In addition, Rayleigh streaming is caused by Reynolds stresses [5] due to the difference of viscosity of gas parcel through the pulse tube in the boundary layer. Moreover, it was demonstrated that there is a third type streaming in PTR. We firstly pointed out in theory [6] that pressure drops due to eddies at entrances and exits in regenerator are asymmetrical with the direction of flow which causes the third type of DC gas flow in PTR. Then we discovered and verified [7] this new type streaming in PTR through a series of experiments. We also introduced a preliminary suppression method [8] in the coaxial PTR by adjusting the metering valve which connects the reservoir to the back pressure chamber of the compressor.

However, until now the physical mechanism of the formation for the third type streaming in the PTR is not clear and the experimental observation is quite difficult. There are several interesting investigations on nonlinear effects besides Gedeon and Rayleigh streaming in PTR and other thermoacoustic engines which use computational fluid dynamics (CFD) method in most cases. Mironov et al. [9] showed in theory that the differentially heated stack provides a source of streaming additional to Rayleigh streaming in

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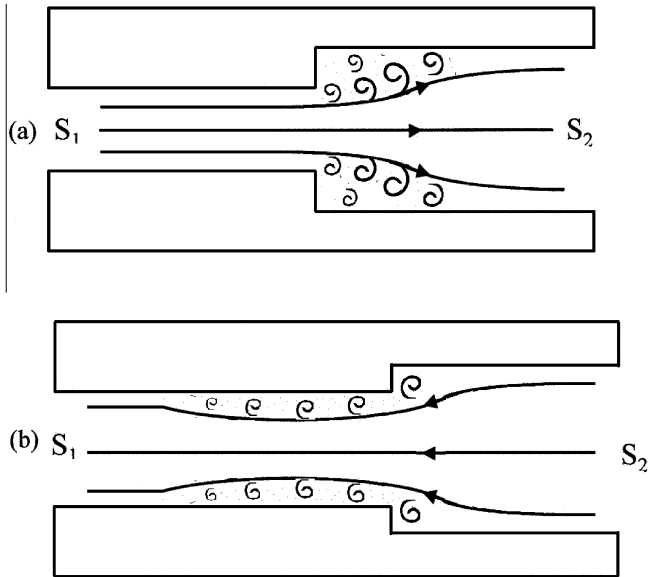


Fig. 1. Qualitative picture of streamlines through a sudden flow area enlargement from S_1 to S_2 , (a) inflow, (b) outflow.

Table 1
Geometry details and boundary conditions of all the cases.

Components	Diameter (mm)	Length (mm)	Wall thickness (mm)	Wall material	Boundary condition
Aftercooler	26.1	20	0.2	Cu	300 K
Regenerator	26.1	70	0.2	Stainless steel	Adiabatic
Cold heat exchanger(CHX)	26.1	10	0.2	Cu	Adiabatic
Pulse tube	11.4	70	0.2	Stainless steel	Adiabatic
Hot heat exchanger(HX)	11.4	10	0.2	Cu	300 K
Inertance tube(IT)	2	1000	0.2	Cu	Adiabatic
Reservoir	60	100	0.5	Stainless steel	Adiabatic

quasi-unidirectional gas flow. Flake and Razani [10] firstly used the FLUENT commercial CFD package and obtained recirculation patterns in their simulated pulse tube. Cha et al. [11] confirmed that CFD simulations such as FLUENT are capable of studying complex periodic processes in PTR and discovered significant multi-dimensional flow effects. Liang and de Waele [12] numerically analyzed a new type of streaming which is caused by the asymmetrical stresses due to the flow resistance of the straightener. More recently, Antao and Farouk [13] reported cycled averaged secondary flow in the pulse tube and concluded that it is actually Rayleigh streaming [14] and it can be suppressed by tapered angle pulse tube.

In the present paper, we firstly discuss the formation mechanism of the third type streaming. The theory is inspired by the applications of jet pumps [15–17] to suppress Gedeon streaming in a traveling wave thermoacoustic engine. We use this simplified theory to calculate the pressure drops due to minor losses along the PTR. Then we show our CFD results to describe the heat and

Table 2
Parameters of porous media.

Components	Mesh number	Parameters	Value
Regenerator	400	ε	0.692
		$\bar{\beta}$	3.953×10^{10}
		$\bar{C}$	120000
Aftercooler, CHX and HX	325	ε	0.68
		$\bar{\beta}$	7.435×10^8
		$\bar{C}$	8147

Table 3
Time step for all cases.

Case	Frequency (Hz)	Time step (s)
1	60	3.3333×10^{-4}
2	45	4.4444×10^{-4}
3	30	6.6667×10^{-4}

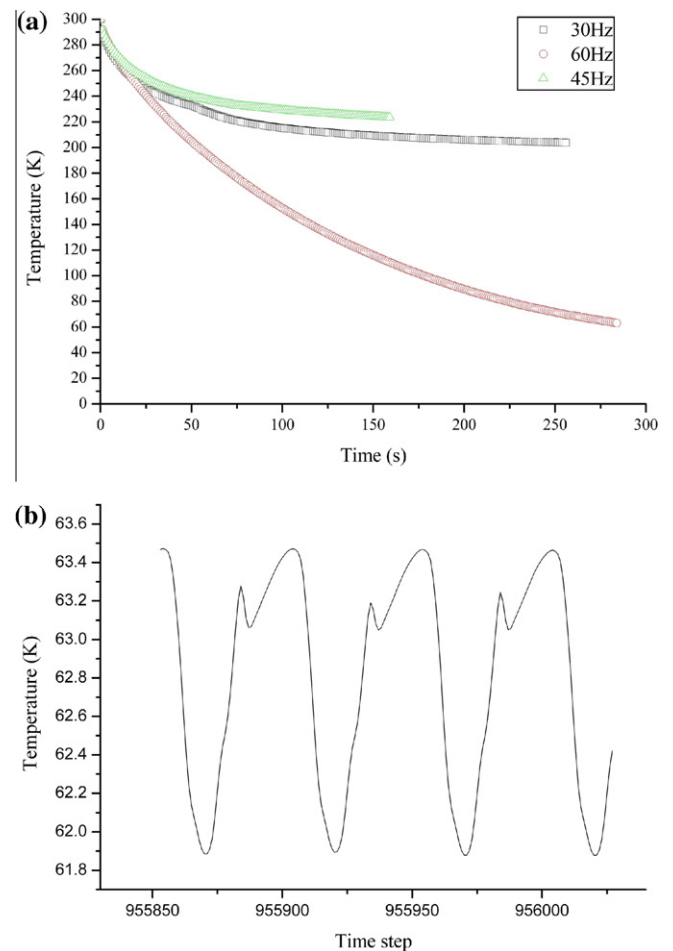


Fig. 3. (a) CFD results of the cooling temperature at the exit of CHX. (b) Variations of the facet average temperature at the exit of CHX after steady state for 60 Hz.

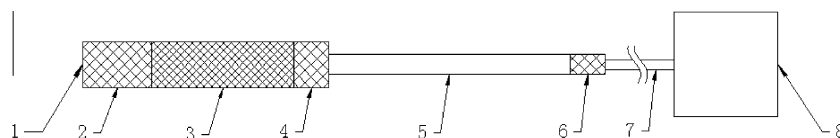


Fig. 2. Schematic of the inline PTR: (1) pressure inlet (2) aftercooler (3) regenerator (4) cold heat exchanger (5) pulse tube (6) hot heat exchanger (7) inertance tube (8) reservoir.

fluid asymmetries to find out the most important factors influencing the streaming.

2. Theory

Sudden enlargement and contraction of tube cross sections exist between the regenerator, heat exchangers, pulse tube and inertance tube (or orifice) in PTR which generate pressure drops due to asymmetrical minor losses. Minor losses are physically associated with the formation of vortices with additional dissipation of energy in them. The generation, evolution and shedding of vortices are induced by the asymmetry of the hydrodynamic effects and the temperature gradients. From Helmholtz equation, we can get the physical mechanism of formation and evolution of vortices.

$$\frac{d\omega}{dt} = (\omega \cdot \nabla)v - \omega(\nabla \cdot v) + \frac{1}{\rho^2} \nabla \rho \times \nabla p + \nu \nabla^2 \omega + \nabla \times f \quad (1)$$

where ω is vorticity, v is velocity, ρ is density, p is pressure, ν is viscosity and f is body force. For high frequency oscillating flow in the PTR, the inflow and outflow alternate rapidly in a period and change the flow pattern dramatically in the transition areas. Moreover, the temperature gradient in the regenerator and pulse tube will significantly change the viscosity and density of working gas in PTR. Thus

the term $\frac{1}{\rho^2} \nabla \rho \times \nabla p$ will be nonzero. Due to the viscosity of fluid, non-slip wall boundary condition and adverse pressure gradient, the boundary layer will be separated near the enlargement and contraction positions. Consequently, the first vortex is generated with spread of vorticity. Furthermore, the evolution process of vortices for oscillating flow in PTR is totally different from that of steady flow. Firstly, one or more vortices may be induced by the adverse pressure gradient attributed to the flow deceleration half period and interactions between the oscillating fluid and wall. Secondly, the flow regime is different from vortex street which is recognized as vortex wave [18]. The main oscillating flow passes around the vortices which are generated periodically so that the local heat and mass transfer is enhanced. Finally, the vortex type is determined by Strouhal number (St) and temperature gradient. This process which transfers momentum from oscillatory flow to DC flow can be observed in the form of circulation patterns and quantitatively verified with the help of CFD tools.

Here we will focus on the DC pressure drops mainly due to minor losses in PTR. Minor losses have been well studied for steady flows. However, for oscillating flows the assessment of the pressure difference is not well understood. Since there is no analytical model for minor losses in unsteady flows at present, we will calculate the minor losses in oscillating flow by cycle averaging the steady flow relations. The pressure drop for steady flow is

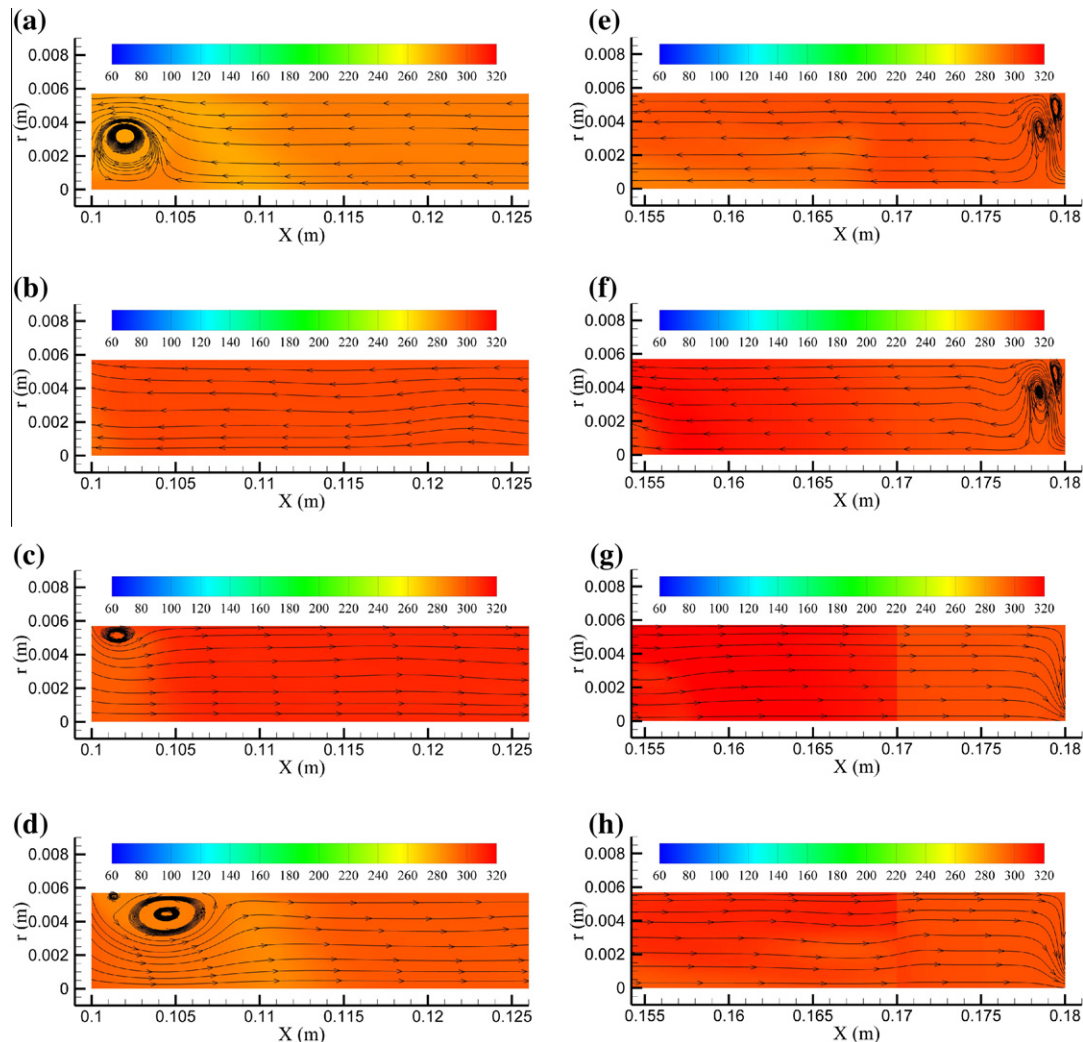


Fig. 4. The temperature contour and streamlines at the beginning of flow time for 60 Hz: (a)–(d) near the cold end of pulse tube at every 1/4 cycle phase $\Phi_1, \Phi_2, \Phi_3, \Phi_4$. (e)–(h) near the exit of hot heat exchanger at $\Phi_1, \Phi_2, \Phi_3, \Phi_4$.

$$\Delta p_{in}^{ml} = \frac{1}{2} K_{in} \rho u^2 \quad (2)$$

$$\Delta p_{out}^{ml} = -\frac{1}{2} K_{out} \rho u^2 \quad (3)$$

where K is the minor loss coefficient, and u is the velocity. The definition of inflow and outflow is shown in Fig. 1. Moreover, we consider a sinusoidal fluctuation of the velocity,

$$u = u_1 \sin \omega t \quad (4)$$

Hence we get [9]

$$\overline{\Delta p^{ml}} = \frac{\omega}{2\pi} \left(\int_0^{\frac{\pi}{\omega}} \Delta p_{out}^{ml} - \int_{\frac{\pi}{\omega}}^{\frac{2\pi}{\omega}} \Delta p_{in}^{ml} \right) = -\frac{1}{8} (K_{out} - K_{in}) \rho u_1^2 \quad (5)$$

For oscillating flow in PTR, ideal gas assumption is made, $p/\rho = RT$. The coefficients K_{in} and K_{out} depend on the geometrics as well as Reynolds number. Hence the time averaged minor loss pressure drop along the pulse tube should depend on geometrics, Reynolds number, temperature and frequency simultaneously.

Eq. (4) holds for any abrupt flow area changes. Thus the minor loss for junctions between the regenerator, heat exchangers, pulse tube and inertance tube can be calculated. Take the pulse tube for instance, the asymmetry pressure drops along the hot and cold end of the pulse tube is:

$$\overline{\Delta p_h^{ml}} - \overline{\Delta p_c^{ml}} = \frac{1}{8} (K_{out,c} - K_{in,c}) \rho_c u_{1c}^2 - \frac{1}{8} (K_{out,h} - K_{in,h}) \rho_h u_{1h}^2 \quad (6)$$

We will use Eq. (6) to calculate the asymmetrical DC pressure drops in pulse tube with CFD simulation results in Section 4.

If we assume the asymmetry flow effect is canceled due to the symmetry flow area change at both ends, thus $K_{out,h} - K_{in,h} = K_{out,c} - K_{in,c}$. Then we get

$$\overline{\Delta p_h^{ml}} - \overline{\Delta p_c^{ml}} = \frac{1}{8} (K_{out} - K_{in}) \left(\frac{\rho_c}{RT_c} u_{1c}^2 - \frac{\rho_h}{RT_h} u_{1h}^2 \right) \quad (7)$$

This equation indicates that even though the fluid flow is symmetrical, the pressure drops will still be induced by the temperature and pressure difference.

Furthermore, the nonlinear effect including the formation of vortices and power dissipation is accompanied by the nature of oscillating flow in PTR and other thermoacoustic systems. The form for the loss of acoustic power $\Delta \dot{E}$ derived by Swift and Ward [19] is

$$\Delta \dot{E} = -\frac{4}{3\pi} \frac{\rho m^4}{2} K |u|^3 \quad (8)$$

where A is the cross sectional area.

From our previous analysis, the formation of accumulated pressure drop is similar to Gedeon flow. However, if there is no closed-loop in PTR which is the key structure for Gedeon flow,

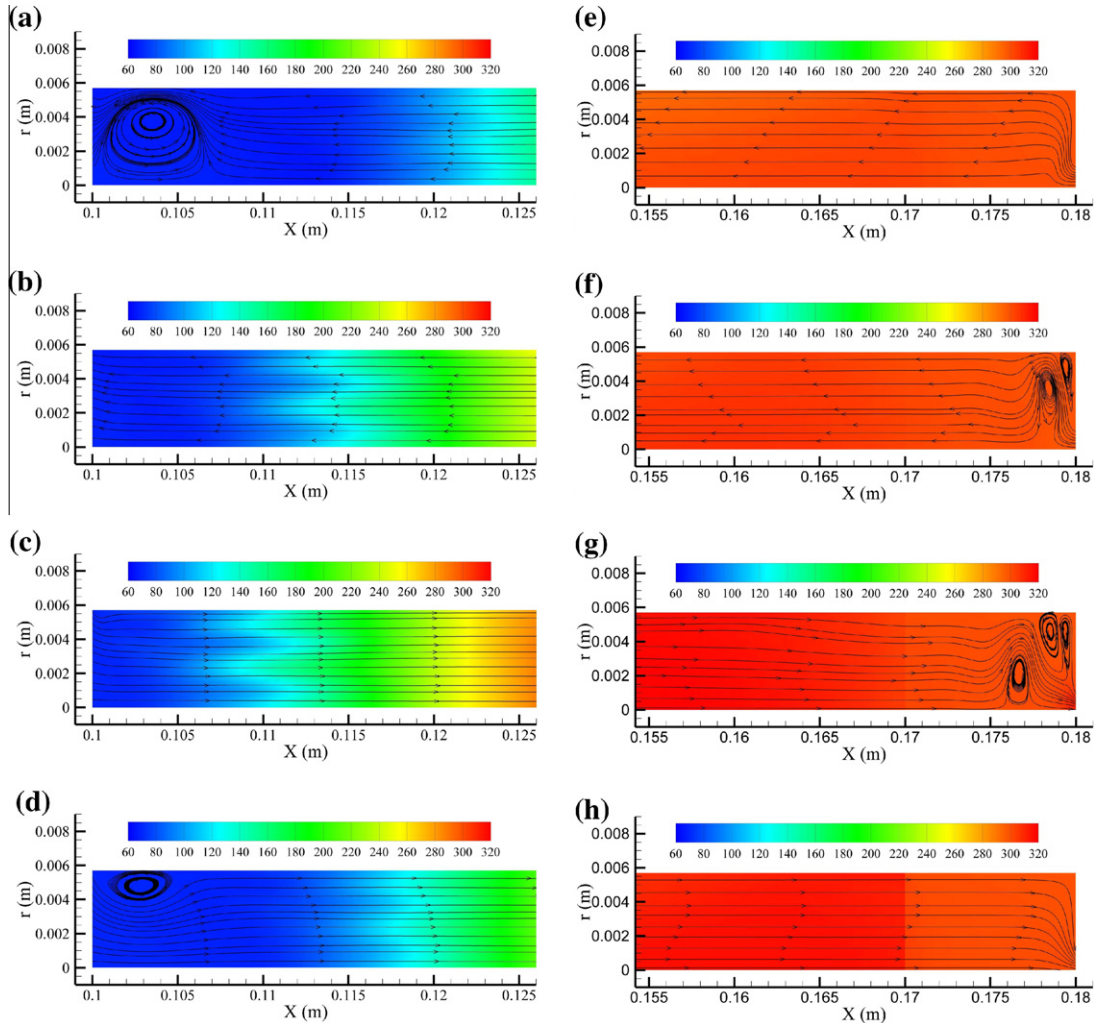


Fig. 5. The temperature contour and streamlines after steady state for 60 Hz: (a)–(d) near the cold end of pulse tube at every 1/4 cycle phase $\Phi_1, \Phi_2, \Phi_3, \Phi_4$. (e)–(h) near the exit of hot heat exchanger at $\Phi_1, \Phi_2, \Phi_3, \Phi_4$.

the minor loss pressure drop still exists and will generate the third type streaming. The time averaged mass flow rate of the streaming is

$$\dot{M}_2 = C \left(\rho_0 \Delta p_0 + \frac{1}{2} (\rho_1 \Delta p_1 \cos \alpha) \right) \quad (9)$$

where C is the geometrical constant, α is the phase angle between pressure drop and density, subscripts 0 and 1 represent the average and first order value. This Equation is appropriate for both Gedeon and the third type streaming but has different physical significance. Furthermore, for an approximate estimation, the same constant C is assumed for the AC flow hence the time averaged amplitude of AC flow rate is $\dot{m} = C \rho_0 \Delta p_1$. To eliminate C , we can divide the DC flow magnitude by AC flow amplitude. Then the mass flow rate of the streaming will be predicted combined with DC and first order pressure drops as well as AC flow amplitude. Typical scale of the nonlinear effect related to minor loss is also shown characterized by vorticity magnitude.

3. CFD modeling features

FLUENT is used to model the nonlinear fluid flow and heat transfer interaction in an entire inline type PTR for its robustness and precision. This numerical experiment gives a systematical analysis of the pressure drops as well as related vortices and streaming inside the PTR, which significantly affects the refrigeration performance. The simulation method is similar to the previous research such as [11,20,21] which is proved to be quite accurate. Nevertheless, these issues are addressed here for the first time with the help of CFD analysis.

3.1. Simulated systems

Axi-symmetric, two-dimensional flow was assumed everywhere in view of the axi-symmetric inline version and boundary conditions. The geometry parameters are based on our experimental U-type PTR and details are shown in Table 1. The experimental U-type configuration is turned into an axisymmetric two-dimensional model as depicted in Fig. 2 for convenience and calculation efficiency. No double-inlet configuration is used to avoid Gedeon streaming. Mesh generation is performed using GAMBIT[®]. The junction regions of the components are represented with finer meshes for better numerical resolution. The number of grid points is 13339. Grid number independent test is conducted by increasing the grid number to 18527 in case 3. It shows that there is no significant improvement in the precision of the numerical simulation comparing with 13339 points.

3.2. Governing equations

Compared with one-dimensional models, this work utilizes fully nonlinear two-dimensional conservation equations. The transient continuity, momentum and energy equations solved by FLUENT are:

$$\frac{\partial \rho_f}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho_f v_r) + \frac{\partial}{\partial x} (\rho_f v_x) = 0 \quad (10)$$

$$\frac{\partial}{\partial t} (\rho_f \vec{v}_x) + \nabla (\rho_f \vec{v} \vec{v}) = -\nabla p + \nabla (\bar{\tau}) \quad (11)$$

$$\frac{\partial}{\partial t} (\rho_f E) + \nabla \cdot (\vec{v} (\rho_f E + p)) = \nabla \cdot (k_f \nabla T + (\bar{\tau} \cdot \vec{v})) \quad (12)$$

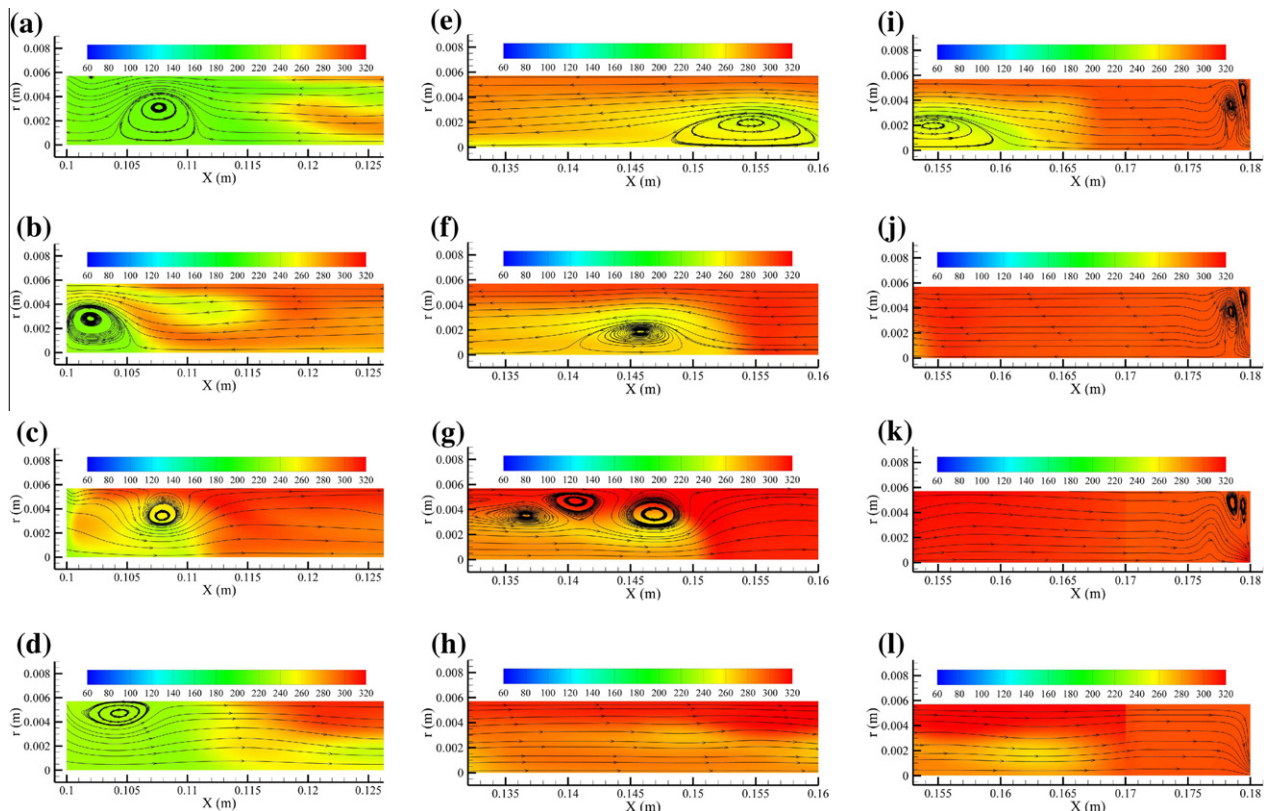


Fig. 6. The temperature contour and streamlines after steady state for 45 Hz: (a)–(d) near the cold end of pulse tube at every 1/4 cycle phase Φ_1 , Φ_2 , Φ_3 , Φ_4 . (e)–(h) in the middle of the pulse tube at Φ_1 , Φ_2 , Φ_3 , Φ_4 . (i)–(l) near the exit of hot heat exchanger at Φ_1 , Φ_2 , Φ_3 , Φ_4 .

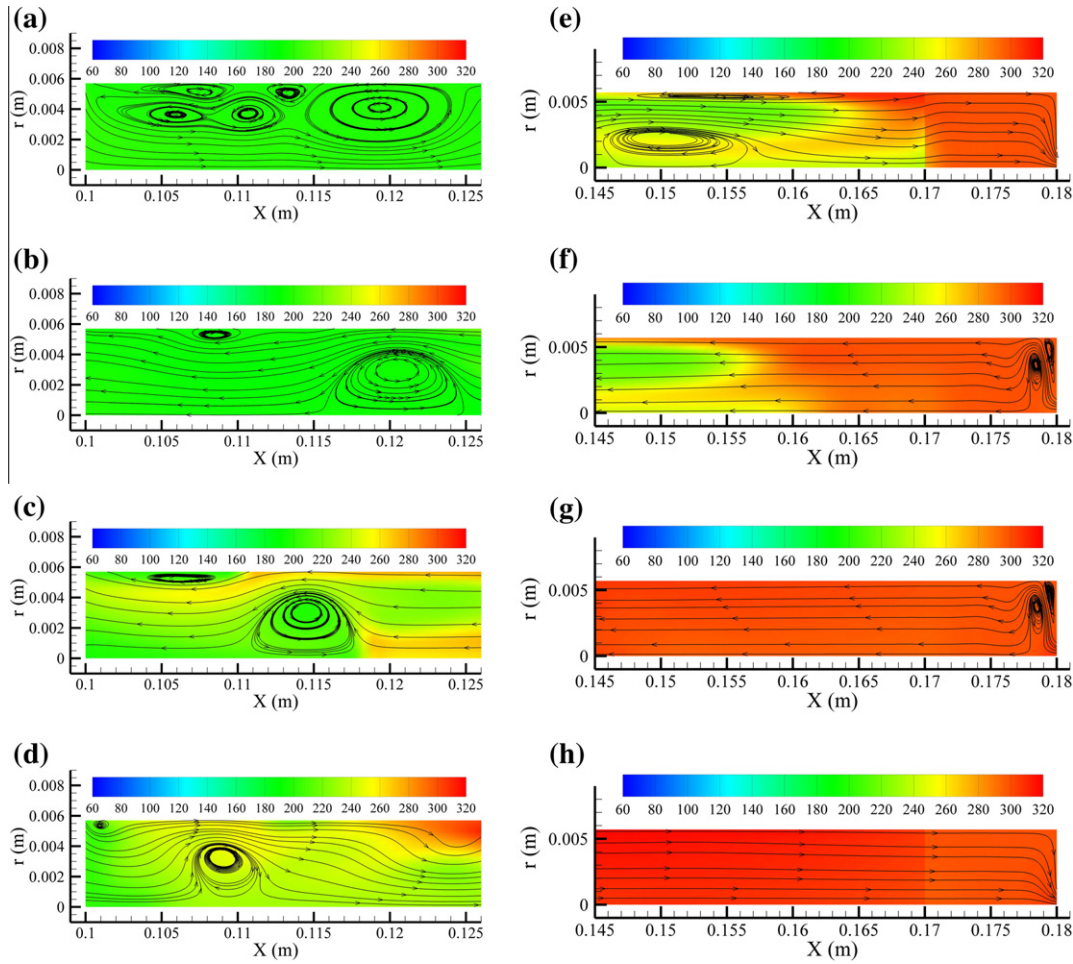


Fig. 7. The temperature contour and streamlines after steady state for 30 Hz: (a)–(d) near the cold end of pulse tube at every 1/4 cycle phase $\Phi_1, \Phi_2, \Phi_3, \Phi_4$. (e)–(h) near the exit of hot heat exchanger at $\Phi_1, \Phi_2, \Phi_3, \Phi_4$.

$$E = h - \frac{p}{\rho} + \frac{v^2}{2} \quad (13)$$

$$\bar{\tau} = \mu \left((\nabla \vec{v} + \nabla \vec{v}^T) - \frac{2}{3} \nabla \cdot \vec{v} \vec{I} \right) \quad (14)$$

All parameters in above equations represent the property of the working fluid helium and ideal gas assumption is made all the time. The above equations apply to the pulse tube, inertance tube and reservoir. Besides, the aftercooler, cold heat exchanger, hot heat exchanger and regenerator are modeled as porous media, assuming that there is local thermodynamic equilibrium between the fluid and the solid structure in these components. The governing equations for the latter four components are:

$$\frac{\partial}{\partial t}(\varepsilon \rho_f) + \frac{1}{r} \frac{\partial}{\partial r}(\varepsilon r \rho_f v_r) + \frac{\partial}{\partial x}(\varepsilon \rho_f v_x) = 0 \quad (15)$$

$$\frac{\partial}{\partial t}(\varepsilon \rho_f \vec{v}) + \nabla \cdot (\varepsilon \rho_f \vec{v} \vec{v}) = -\varepsilon \nabla p + \nabla \cdot (\varepsilon \bar{\tau}) - (\mu \bar{\beta} \cdot \vec{j} + \frac{1}{2} \bar{C} \rho_f |\vec{j}| \vec{j}) \quad (16)$$

$$\begin{aligned} \frac{\partial}{\partial t}(\varepsilon \rho_f E_f + (1 - \varepsilon) \rho_s E_s) + \nabla \cdot (\vec{v}(\rho_f E_f + p)) \\ = \nabla \cdot ((\varepsilon k_f + (1 - \varepsilon) k_s) \nabla T + (\varepsilon \bar{\tau} \cdot \vec{v})) \end{aligned} \quad (17)$$

where ε is the porosity and $\bar{\beta}$ and $\bar{C}$ are the viscous and inertial resistance coefficient tensors according to FLUENT's notations [22]. Subscripts f and s represent fluid and solid respectively.

3.3. Initial and boundary conditions

The boundary conditions of all the components are also shown in Table 1. Table 2 shows the porosity, viscous and inertial resistance coefficients of the regenerator and other porous media according to experimental results in our U-type PTR and literature [23]. For all the three simulated cases, frequency is the only variable and is shown in Table 3. The mean pressure is given as 2.3 MPa. The inlet pressure oscillation is modeled as a sinusoidal function to simulate the effect of piston motion of the traditional compressor or pressure wave generator, $p = p_1 \sin \omega t$, where p_1 is the magnitude of the inlet pressure and given as 0.25 MPa. A simple user defined function (UDF) is used to determine the inlet pressure according to the different operating frequencies for these three cases. In addition, the viscosity, thermal conductivity and specific heat capacity of helium are taken to be temperature dependant. UDFs are also written to calculate these variable thermo-physical properties. For simplicity, all the simulations started with an initial system temperature of 300 K and axial velocity of 0.1 m/s.

3.4. Method of solution

Transient and compressible flow mode is used to simulate the PTR system and proper method is crucial to a successful simulation. Consequently, the scheme based on segregated solver, second-order implicit time, second order upwind spatial differencing and SIMPLE algorithm are chosen. PRESTO! is used as the pressure

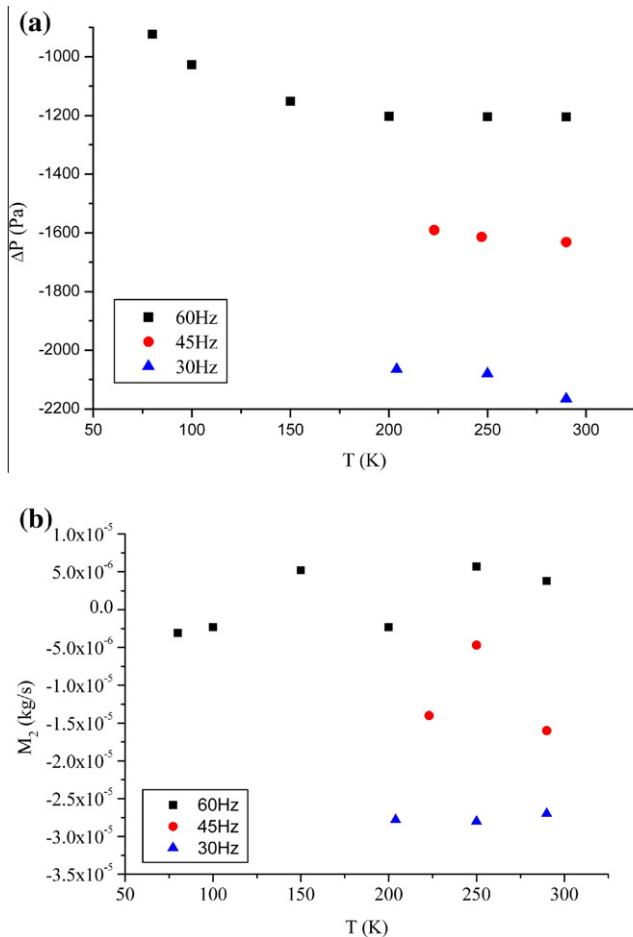


Fig. 8. (a) The asymmetry DC pressure drops between the cold end of pulse tube and the exit of hot heat exchanger due to minor losses for different CHX exit temperature. (b) The predicted mass flow rate of the DC flow for different CHX exit temperature.

discretization method as a result of the porous media model used in regenerator and other heat exchangers. The fixed time step is changed for different frequencies in three cases shown in Table 3. At each time step, 50 inner iterations are used for proper coupling between equations and better convergence. The absolute convergence criterion is 10^{-4} for the continuity, axial and radial velocities, while it is 10^{-6} for energy. Furthermore, considering the main research object is the mechanism of the streaming without turbulence, laminar mode is chosen to simulate the whole system as the Reynolds number is small in most components in the system except in the inertance tube.

4. Results and discussion

4.1. Cooling temperature

Simulations were continued until steady periodic state was obtained. The criterion for steady periodic condition is that the facet average temperature of the cold heat exchanger (CHX) will reach a steady state. The variations of the facet average temperature at the exit of the CHX are depicted in Fig. 3, and the temperature of steady periodic state for case 1 is also shown. The deviation from exact sinusoidal temperature variation over one cycle is due to the complex interaction between the mesh and gas in porous media components. Since the lowest cooling temperature of the PTR is important for practical applications, the experimental results are

Table 4

Convective heat transfer loss and acoustic power loss for all cases.

Case	Frequency (Hz)	Heat loss (W)	Acoustic power loss at cold end (W)	Acoustic power loss at hot end (W)
1	60	1.6	0.001	0.5
2	45	7.3	0.0011	1.2
3	30	14.5	0.002	1.7

also shown to validate the simulations and will be discussed in Section 4.3.

4.2. Nonlinear processes: streaming induced by vortices

The numerical simulations revealed the mechanism of the third type streaming in PTR. Figs. 4 and 5 show the temperature gradient and streamlines between the cold end of pulse tube and the exit (right end as shown in Fig. 2) of hot heat exchanger for 60 Hz at different time. It's clearly observed that vortices are generated near the hot and cold ends where the sudden enlargement and contraction exist. This phenomenon occurs from the very beginning of the cooling process to the final steady periodic state. The vortex pattern shown in Figs. 4–7 is in accordance with our theoretical analysis in Section 2. The vortices are generated and vanished periodically. Moreover, for every period, the flow regime changes with phase of the main oscillating flow. At the acceleration phase of inflow from the cold end, one vortex is generated near the cold end of pulse tube due to minor losses. The following evolution process is much more complicated due to different Strouhal number and temperature gradient. But there are some common characteristics in principle. At the deceleration phase of inflow, the first vortex maybe exists or disappears according to the pressure gradient and vorticity dissipation rate, but other vortices may be induced attributed to the adverse pressure gradient in the middle of the pulse tube caused by interactions between the following inflow and wall. The outflow condition is similar for the first half period of the other end and vice versa for the next half cycle.

Fig. 4 shows the flow field with almost no temperature gradient from which we can conclude that the streaming is induced by vortices due to minor losses and it is totally different from Rayleigh streaming. Furthermore, compared with Fig. 5, the thermodynamic asymmetry in the pulse tube also contributes to the scale, shape and position of the vortices.

The evolution processes of the streaming for different frequencies are quite different which is the most crucial and interesting part. Figs. 4 and 5 show that in the 60 Hz PTR vortices only occur at the end of the pulse tube at a certain instant in one cycle. However, in the 45 Hz and 30 Hz PTR shown in Figs. 6 and 7, the vortices formed at one end of the pulse tube gradually spread to the middle region and become much larger in a period. The reason of the difference lies in that vortex numbers and patterns are dependent on $St = fd/u$, where f is oscillating frequency, d is diameter and u is velocity. CFD results after steady state show that St is 0.38, 0.19 and 0.11 for the 60, 45 and 30 Hz case respectively. It's revealed that the larger St leads to fewer vortices. Several vortices transport energy one by one from cold to hot end which cause strong mix and coupling of fluid and heat flow. In the macro angle of view, with the formation, evolution and shedding of vortices, the DC flow from one end to another in the PTR is formed in a cycle accompanying alternating main flow. This nonlinear process will dramatically deteriorate the refrigeration performance. As a result, the lowest temperature for cases 2 and 3 are higher than case 1.

While there is asymmetrical pressure drops, there may be DC gas flow. The DC pressure drops due to minor losses and tempera-

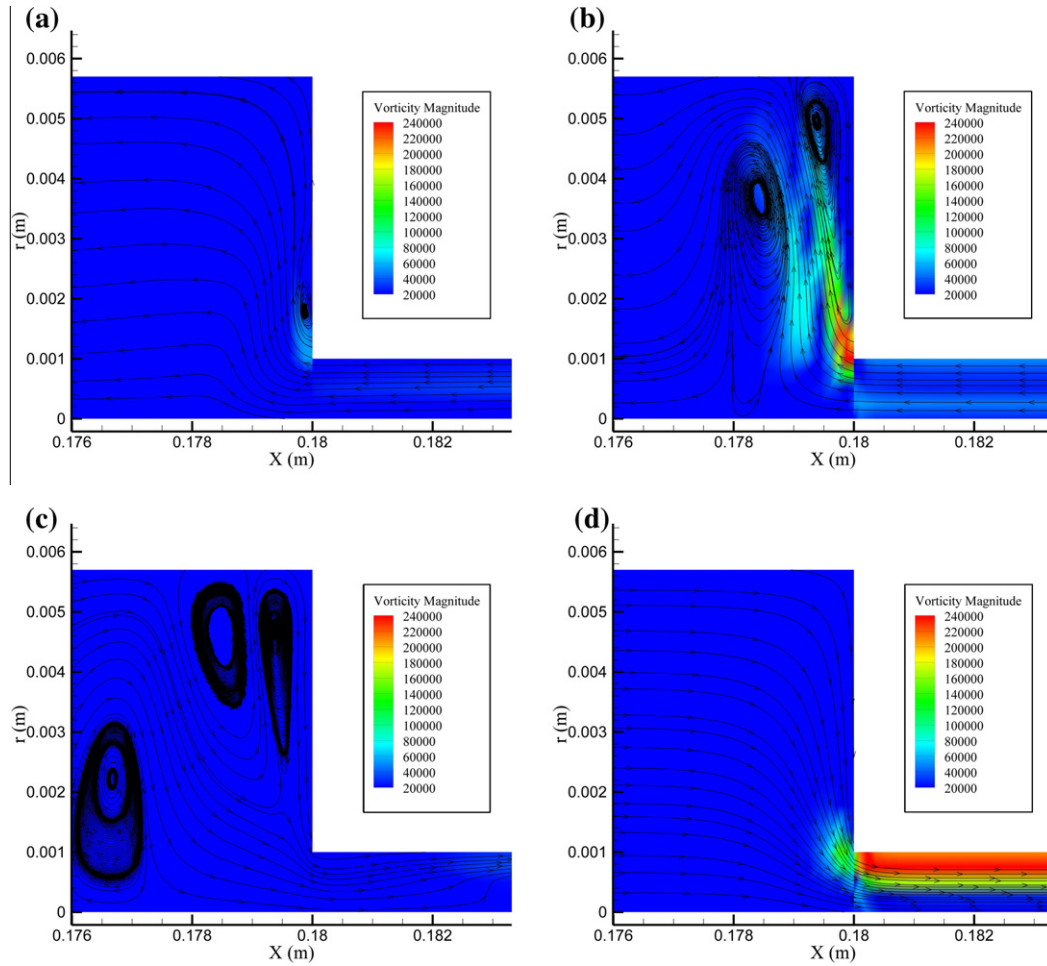


Fig. 9. The vorticity magnitude contour and streamlines after steady state for 60 Hz: (a)–(d) near the exit of hot heat exchanger at every 1/4 cycle phase Φ_1 , Φ_2 , Φ_3 , Φ_4 .

ture gradient have been analyzed in Section 2. Here we assume the coefficients K_{in} and K_{out} depend only on the geometrics.

$$K_{in} = (1 - S_1/S_2)^2 \quad (18)$$

$$K_{out} = 0.5(1 - S_1/S_2) \quad (19)$$

where S_1 and S_2 represent flow area shown in Fig. 1. Thus the DC pressure drops are calculated using Eqs. (6), (18), and (19) shown in Fig. 8(a). It can be seen that the order of magnitude of the DC pressure drop is several kPa. The mass flow rate of the streaming is calculated using DC and first order pressure drops as well as AC flow amplitude derived in Section 2. Fig. 8(b) shows the CFD results of the value. The DC flow is obviously very sensitive to frequency as both the magnitude and direction of the streaming are changed with frequency. It seems that for 30 and 45 Hz cases, the mass flow rate of the streaming is on the order of 10^{-5} kg/s. But for 60 Hz, the mass flow rate is on the order of 10^{-6} kg/s for vortices shown in Fig. 5 are smaller and fewer than the other cases. This phenomenon is important for the design of PTR when choosing the proper coupling of geometry and frequency and needs more research for other geometries. Moreover, the change of temperature distribution inside the system also affects the streaming. It may be the main reason why DC gas flow is highly erratic and unstable as observed in the experiments [7,24] before.

Though the absolute value of DC flow rate is relatively small compared with the AC flow rate in the pulse tube, the induced acoustic power dissipation and convective heat transfer may be

significantly detrimental to refrigeration performance. The heat transfer loss can be predicted by $Q_{loss} = \dot{M}_2 c_p (T_h - T_c)$, where c_p is specific heat capacity, T_h and T_c is hot and cold end temperature respectively. If T_h is 300 K and the vortex length is approximately 1/3 of the pulse tube, then T_c is assumed to be 200 K for estimation. Table 4 shows the results of heat loss as well as acoustic power loss calculated from Eq. (8). Compared with hot end, the acoustic power dissipation at cold end due to minor losses can be neglected. However, the heat loss of the streaming induced by DC pressure drops is considerable. With the similar mechanism in our previous experimental research [7], the heat loss was 2 W at 59 Hz which partly reinforces the present results.

The vorticity magnitude at the exit of the hot heat exchanger of different phases in a cycle is shown in Fig. 9. The large vorticity is generated at the junction between the hot heat exchanger and the inertance tube, which spread around the system with the oscillating flow. Diffusion and dissipation of vorticity from the junction region to the inertance tube is significant. Hence the vorticity magnitude should also be considered as a good parameter to characterize the nonlinear processes in the PTR.

4.3. Experimental validation

Preliminary comparative experiments are made on a U-type PTR with the similar geometry of the main components as the simulated inline PTR in order to evaluate validity of the CFD simulation. Boundary and initial conditions such as the operating frequency, mean pressure and dynamic pressure at the inlet of

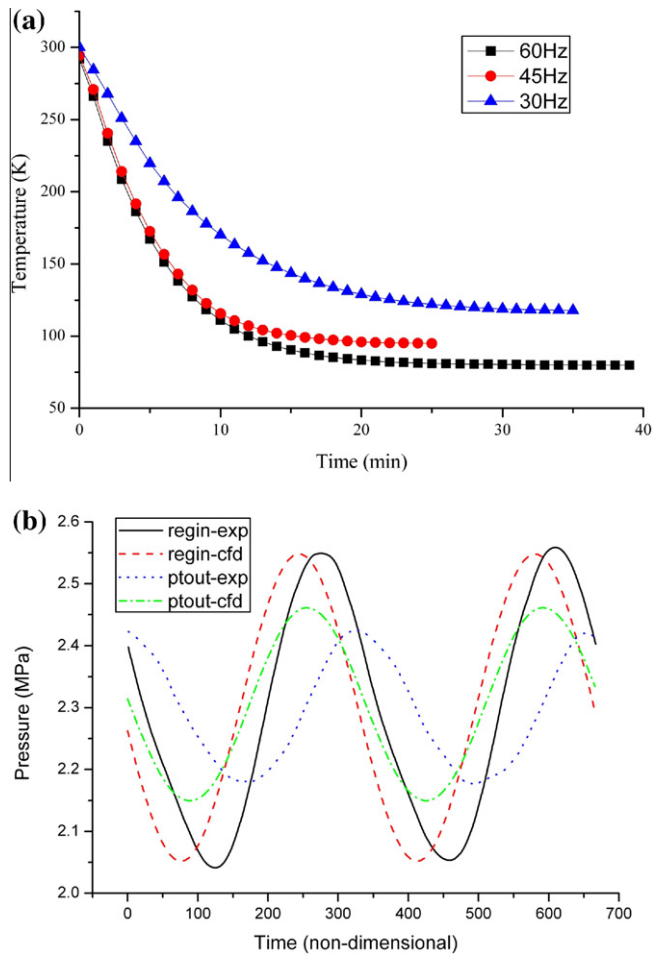


Fig. 10. (a) Experimental results of cooling temperature for all cases. (b) The experimental and CFD results of pressure wave at cold end of regenerator and hot end of pulse tube for 60 Hz.

the regenerator are controlled the same as the simulated cases. As shown in Fig. 10(a), the lowest refrigeration temperature for the 60 Hz case is in good agreement with the calculated values. But for the 45 and 30 Hz cases, the discrepancy between the simulated and experimental results is relatively large. It is mainly due to two reasons. Firstly, we use laminar model to simulate the whole system and the inertance tube is too sensitive for frequency change. So if the inertance tube is not optimized for the specific frequency, the refrigeration performance is much worse than experiments. Secondly, St may be different for the simulation and experiment as a result of the difference between U-type and inline configuration. Moreover, flow-straighteners were used in experiments which greatly help to reduce the irreversible entropy production due to vortices [25]. Hence in some way the effects of the third type streaming in the experiments for 30 and 45 Hz are much less than in CFD modeling. This result also proves the validity of using flow-straighteners to suppress the streaming.

Furthermore, the experimental and simulated results of the pressure at inlet of the regenerator and exit of the pulse tube for 60 Hz are presented in Fig. 10(b). Though the phase lag has some differences, the amplitude of DC and first order pressure drops are quite precise. As a result, the value of streaming estimated in Section 4.2 is accurate. Based upon above comparisons, the CFD simulation reasonably describes the operating mechanism and performance of the PTR. However, better models need to be established to predict the phase shift in the regenerator more precisely.

5. Conclusions

This paper numerically examined the nonlinear and multidimensional transport phenomena in PTR. From the CFD analysis, vortex formation and evolution are illuminated and it can be concluded that the physical nature of the third type streaming in PTR is vortices caused by pressure drops resulting from fluid and heat asymmetries. The streaming is found to be very sensitive to frequency change and the process is controlled by Strouhal number and temperature gradient. The nonlinear effects could seriously deteriorate the overall performance of the system. The CFD study is validated by experiments and our next step will focus on the mechanism of pressure drops and related streaming in the regenerator by experiments as well as using better numerical model for PTR.

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